Sets and Set Operations

Set theory is a branch of mathematics, which deals with the properties of well-defined collections of objects, which may or may not be of a mathematical nature, such as numbers or functions. It is used for the foundation of various topics in mathematics. Now a days, it is also used in the field of economics also.

George Cantor (1845-1918), a German mathematician is regarded as the founder of 'set theory.' Sets are an organized collection of well-defined distinct objects. The objects may be anything, i.e., a group of numerical numbers, letters, people, animals, countries, etc. The objects constituting a set are termed as its "elements" or "members". No element is repeated in a set, ~~but~~ or no two elements of a set are identical.

The sets are usually denoted by capital letters like A, B, C, etc. and the elements of a set are denoted by small letters like a, b, c, etc. and by numerical numbers.

Examples of set are as follows:

(a) Students of a college/department,

(b) A team of a cricket/football players,

(c) All members of a family

(d) All text books of a library, etc.

Notations/Presentation of a Set:

There are two ways of representing a set -

(a) **Roster Form or Tabular Form:** When all the elements of a set are listed inside the bracket { } and separate the elements with commas (,), (without repeating any of the elements) then this method of describing a set is known as Roster Method or Tabular Method. For example:

1. The set of all odd natural numbers less than 10, ~~is~~ can be expressed as $A = \{1, 3, 5, 7, 9\}$

2. The set of all vowels in the English alphabet, can be expressed as $B = \{a, e, i, o, u\}$, etc.

<u>Note</u>: In roster method, the order in which the elements are listed is immaterial. Thus the 1st example can also be represented as $A = \{9, 7, 5, 3, 1\}$ or $A = \{3, 5, 1, 9, 7\}$, etc.

Similarly, the 2nd example can also be expressed as $B = \{u, o, i, e, a\}$ or $B = \{e, a, i, o, u\}$, etc.

<u>(b) The Rule Method or Set Builder Method</u>: When all the elements consisting a set have a common property and it is possible to represent the set by describing the common property of the elements, then the representation is termed as Set Builder Method.

<u>For example</u>: A is a set of all those even positive numbers which are greater than 2 and less than 12, then it can be expressed in Set Builder form as –

$A = \{x : x$ is a positive even number and $2 < x < 12\}$.

The dots ':' is read as 'such that'.

Similarly, if A is a set of all vowels in the English alphabet, then it can be represented in Set Builder form as –

$A = \{x : x$ is a vowel in the English alphabets$\}$

Note: In this method, the elements of a set are represented by using a symbol 'x', which is followed by a colon ":" After the sign of colon, the common property of the elements of the set is described and enclose the whole description within braces. The symbol "|" is also used instead of ":" in this method.

Many times it is not possible to list all the elements of a set, as when a set ~~contains~~ consists of very large number of elements. In many other cases, it is not necessary to list all the elements, as when they are related to each other or have a common property ~~as~~ and can be expressed ~~with~~ the relation ~~with the help of~~ on the common ~~rod~~ property. in a phase or sentence or symbolically by a rule. In these cases, the Rule Method is very much helpful in representing the sets. Many sets can only be represented by this method only, like - set of all fishes in the Indian Ocean, set of all insects ~~of~~ in the Kaziranga National Park/Meghalaya State, etc. Even if there are finite number of elements in these sets, it is not possible to know their actual number and cannot enumerate them.

All the elements of a set belong to that particular set and this belongingness of an element to a set is represented with the help of the symbol '$\in$' (Greek ~~lett~~ symbol 'epsilon') (read as 'belongs to'). Thus, if a particular element 'p' is an element of a set 'A', it is expressed as $p \in A$. On the other hand, if the element 'p' is not an element of the set 'A', then it is represented as $p \notin A$. For example, if a ~~a~~ represents a group of students of Dibrugarh University (D) and b represents a group of students of ~~Delhi University~~ Gauhati University (G), then $a \in D$, but $a \notin G$ and $b \in G$, but $b \notin D$

Types of Sets:

1. Finite and Infinite Sets: A set is finite, if it contains a definite number of elements or it has not any element. For example,

(a) A is a set of the days in a week, i.e.,
$A = \{$Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$\}$

or $A = \{x \mid x \text{ is a day of a week}\}$

(b) $B = \{x \mid x \text{ is student of a class}\}$

~~On the other hand,~~

(c) $A = \{x : x \text{ is odd number and } x < 12\}$

On the other hand, if the number of elements of a set is very large and infinite, then the set is termed as Infinite Set. For example-

(a) A is a set of all natural numbers, i.e.,
$A = \{x \mid x \text{ is natural number}\}$

(b) $B = \{x \mid x \text{ is fishes of a river}\}$

(c) $A = \{x \mid x \text{ is a point on a straight line } PQ\}$.

2. Null Set or Empty Set: A set which does not has any element or any set without any element is known as Null Set, or Empty Set or Void or Zero Set. and it is denoted by the symbol ϕ (read as phi) and in tabular form it is expressed as $\{\}$.

For example, $A = \{x \mid x$ is a married bachelor$\} = \phi$

$B = \{x \mid x$ is a natural number and $1 < x < 2\}$

3. <u>Unit Set or Singleton Set</u>: A set which has only one element is termed as unit set or singleton set. For example, $A = \{1\}$, $A = \{b\}$, $C = \{x \mid x$ is present Prime Minister of India$\}$

It is noted that if $A = \{0\}$, it means A has one element 0, hence it is not a null set.

4. <u>Universal Set</u>: A universal set is a set which contains all the elements or objects of other sets, including its own elements. There is no formula to find the universal set, it is possible to have to represent all the elements in a single which is collectively termed as universal set. Thus, it is a set which has elements of all the related sets, with out any repetition of elements. It is usually denoted by the symbol 'U' or Ω and it may differ, from context to context. For example, set A consists of all even numbers such that, $A = \{2, 4, 6, 8, ----\}$ and set B consists of all odd numbers, such that, $B = \{1, 3, 5, 7, ----\}$. Then, the Universal set U consists of all natural numbers, such that, $U = \{1, 2, 3, 4, 5, 6, 7, -----\}$

5. <u>Equal Sets or Identical Sets</u>: Two sets are said to be equal, if they contain the same distinct elements, even if the appearance of the elements

of the two sets are not in the same order. For example, two sets A and B having the order of elements as -

$A = \{a, b, p, q\}$ and $B = \{p, q, a, b\}$

then, it can be said that $A = B$., because both the sets contain the same elements, even if the order of the elements are not same.

6. Equivalent Sets: Two sets are said to be equivalent if there is one to one correspondence between the elements of the two sets. Equivalent sets have same number of distinct elements but not the same elements. For example -

$A = \{1, 2, 3, 4\}$ and $B = \{p, q, r, s\}$, then A and B are said to be equivalent sets, since a one-to-one correspondence exists between the elements of A and the elements of B and it is expressed as - $A \equiv B$ or $A \Leftrightarrow B$.

7. Disjoint Sets: If two sets do not have any common element, then they are termed as disjoint sets. For example, $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9, 10\}$, then A and B are said to be disjoint sets because they do not have any common element.

8. Sub Sets: If every element of set A is also element of set B, then set A is ~~termed~~ said to be subset of set B and it is expressed as - $A \subseteq B$ (read as A is subset of B or A is included in B). For example, $A = \{1, 2, 3\}$ and $B = \{0, 1, 2, 3, 4, 5\}$, then A is sub-set of B or symbolically $A \subseteq B$, because

every element of set A are also element of of set B.

A set can have a large number of possible sub-sets depending on the number of elements. The number of sub-sets of a set containing 'n' elements can be calculated by this formula 2^n (where n = no of elements). Thus, if a set having 3 (three) elements, then it has $2^3 = 8$ sub-sets). For example-

$A = \{a, b, c\}$, then sub-sets of A are $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Note: (a) Since null set or empty set (ϕ) has no element, therefore it is a subset of every set.

(b) Every set is also a subset of itself, i.e., $A \subseteq A$.

9. Proper Sub-Sets: If each and every element of a set A are also elements of set B, i.e., if $a \in A$, then $a \in B$ also. But every element of B may not be an element of A or there exists at least one element of B that does not belong to A, then A is said to be proper sub-set of B and B is termed as super-set of A. Symbolically, $A \subset B$ (read as A is proper sub-set of B) and $B \supset A$ (B is super-set of A). Thus, if set A is a sub-set of set B, other than ϕ and B, then A is proper sub-set of B. For example, $A = \{2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$, then A is a proper sub-set of B ($A \subset B$) and B is super-set ~~$(B \supset A)$~~ of A ($B \supset A$).

10. Power Set: Power set is the set of all subsets of a set (say A) and is denoted by $P(A)$. For example, if $A = \{p, q, r\}$, then power set of A is

$$P(A) = \{\phi, \{p\}, \{q\}, \{r\}, \{p,q\}, \{p,r\}, \{q,r\}, \{p,q,r\}\}$$

Set Operations: Mathematical operations on sets can be possible, just as similar to the mathematical operation of numbers like addition, ~~in ter~~ subtraction, multiplication, division, etc. The principal operators on sets involve the union, intersection and complements of sets. [Venn diagrams are named after the English logician John Venn (1834-1883).]

Note: Sets and set operations can be represented by draaing diagrams known as Venn Diagrams. The universal set U or Ω is represented by points within a rectangle and set A, B, C, etc. are expressed by points with circles or ovals draan inside the rectangle. The set A (consisting of points within the circle) draan from the universal set U is expressed with the help of the Venn diagram as (figure: 1) and can be say that $A \subseteq B$.

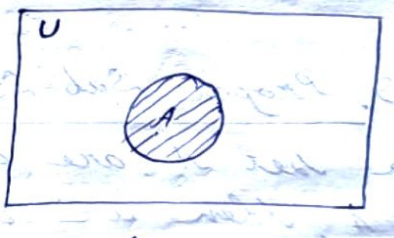

figure: 1

* Venn diagrams help to visually represent the similarities and differences between two groups/concepts or among different groups.

Note: Equal Sets: Two sets A and B are said to be equal only if every element of set A is also element of set B and every element of set B is also element of set A, i.e., A is subset of B and B is subset of A. Thus, symbolically, $A \subseteq B$ and $B \subseteq A$, then $A = B$.

1. Union of Sets: The union of two sets (say set A and B) is the formation of a new set (say set C) consisting of all the elements of the given two sets, avoiding repeation of the elements.** It is symbolized as $A \cup B$. For example,

$$A = \{1, 2, 3\} \text{ and } B = \{3, 4, 5\}$$

$$\text{then } A \cup B = C = \{1, 2, 3\} \cup \{3, 4, 5\}$$

$$= \{1, 2, 3, 4, 5\}$$

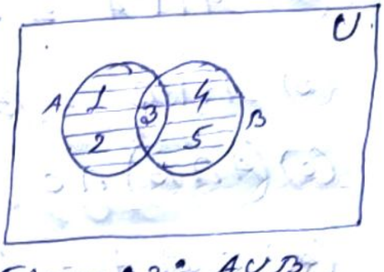

Figure 2: $A \cup B$

Union of A and B ($A \cup B = C$) can also be represented with the help of Venn diagram. It is shown in figure 2, where shaded area represents $A \cup B$.

(** which are either in A or B)

2. Intersection of Sets: The intersection of two sets (say set A and set B) means the formation of a new set (say C) containing all the common elements of the given sets or all the elements which are present in both the sets. It is represented symbolically as $A \cap B$. For example,

$$A = \{1, 2, 3\} \text{ and } B = \{3, 4, 5\}$$

$$\text{then, } A \cap B = C = \{1, 2, 3\} \cap \{3, 4, 5\}$$

$$= \{3\}$$

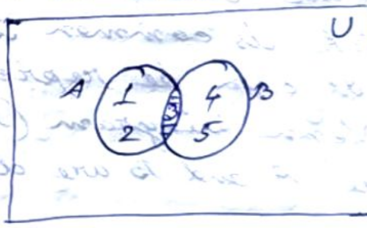

Figure 3: $A \cap B$

Figure 3 represents the intersection of the two sets ($A \cap B$), where the shaded area displays $A \cap B$.

3. Note: Some Properties of the Operation of Union:

If A and B are any two sets and U is the universal set, then -

(a) $A \cup A = A$, (b) $A \cup B = B \cup A$,

Abinash Bharali

(c) $(A \cup B) \cup C = A \cup (B \cup C)$, (d) $A \cup \phi = A$, ~~(e) U∪A~~

(e) $U \cup A = U$

Some properties of Operation of Intersection:

(a) $A \cap A = A$

(b) $A \cap B = B \cap A$

(c) $(A \cap B) \cap C = A \cap (B \cap C)$

(d) $\phi \cap A = \phi$

(e) $U \cap A = A$

(f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

*** Disjoint Sets: If A and B are two sets and they do not have any common element, i.e., $A \cap B = \phi$, then these sets A and B are termed as Disjoint Sets. For example, $A = \{5, 7, 8\}$ and $B = \{10, 11, 12, 13\}$. Thus, set A and B are disjoint sets, because there is no element which is common to A and B. It can be represented by Venn Diagram (Figure: 4) where A and B are disjoint sets.

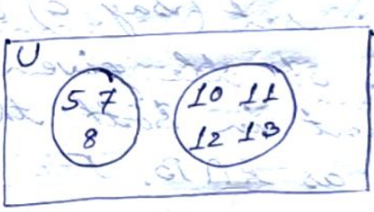

Figure: 4

3. Difference of Sets: The difference of two sets A and B is the set of those elements of A which do not belong to set B. It is represented by the symbol A-B. Thus, symbolically,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

Similarly, the difference between sets B and A can be expressed symbolically as

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

Thus, the difference between sets A and B and

B and A (i.e., $A-B$ and $B-A$) can also be represent-ed with the help of Venn Diagram (Figure: 5 and Figure: 6) as follows:

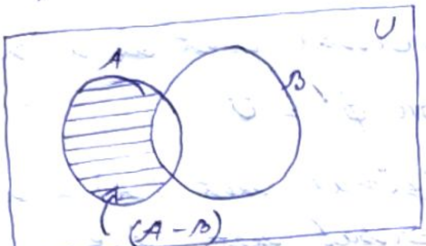

Figure: 5

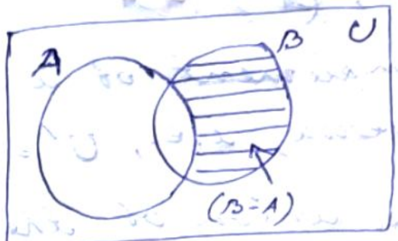

Figure: 6

4. <u>Symmetric Difference of two Sets</u>: If A and B are two sets, then symmetric difference of these sets is the set which contains the elements of either set A or set B, but does not contain those elements which are common to both the sets A and B. It is expressed as $A \nabla B$ and symbolically written as -

$$A \nabla B = \{x \mid x \in A \cup B \text{ and } x \notin A \cap B\}$$

$$= (A \cup B) - (A \cap B)$$

or $A \nabla B = (A-B) \cup (B-A)$

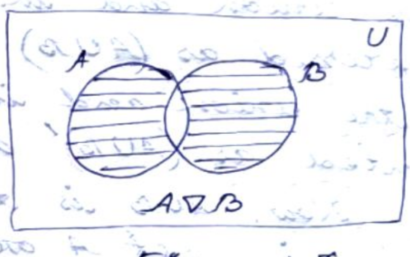

Figure: 7

The symmetric difference of sets A and B is show by Venn diagram (Figure: 7), where the shaded area represents $A \nabla B$.

5. <u>Complement of a Set</u>: If U is the universal set for a set A, then complement of the set A is the set containing all the elements of the universal set U which are not the elements of A. It is denoted as A' or A^c. It is symbolically represented as - A' or $A^c = \{x \mid x \in U \text{ and } x \notin A\}$

or A' or $A^c = U - A$

The complement of a set (say A) can be represented by a Venn diagram (Figure 8), where shaded area represents A' or A^c.

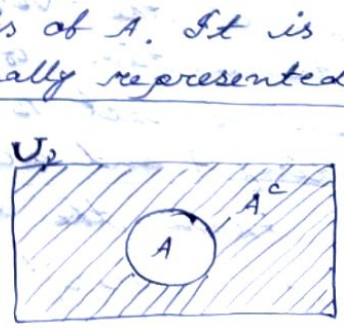

Figure: 8

Complement Laws:

(a) If A' is complement of A, then A is complement of A', i.e., $(A')' = A$

(b) Complement of a universal set is a null set and vice-versa, i.e., $U' = \phi$ or $\phi' = U$

(c) The union of the original set (A) and it's complement (A') is the universal set, i.e.,

$$A \cup A' = U$$

(d) The intersection of the original set (A) and it's complement (A') is the null set, i.e.,

$$A \cap A' = \phi$$ and U is the universal set

De Morgan's Rule: If A and B are two sets, then union and intersection of the two sets can be expressed as $(A \cup B)$ and $(A \cap B)$. Accordingly, complements of the union and intersection of sets A and B is represented as $(A \cup B)'$ and $(A \cap B)'$ respectively.

Now, $A \cup B$ is the set containing all the elements either in set A or set B, without repeation of the elements. It is shown in the figure: 9 and represented by the shaded area. Thus, $(A \cup B)'$ is the non-shaded area of U in the figure: 9.

Figure: 9

Complement of set A is represented as A' is obtained by deleting the elements of set A from U, which gives all the area outside A. Similarly, complement of set B (B') is the area outside set B. Accordingly, the non-shaded area of the Venn diagram (Figure: 9) is representing intersection of the complement of set A (A') and complement of set B (B') or intersection of the area excluding A and area excluding B, i.e., $(A' \cap B')$.

From the above description, it can be concluded that $(A \cup B)' = A' \cap B'$ —— (I)

Thus, equation (I) is termed as De Morgan's rule.

This rule ~~as~~ can be more clearly explained by taking an example.

Let, $A = \{1, 2, 3, 4, 5, 6\}$ ~~and~~, $B = \{4, 5, 6, 7, 8\}$

and, $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Then, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

and $(A \cup B)' = \{0, 9, 10\}$

$= \{$elements of U not in $A \cup B\}$ —— (1)

Now, $A' = \{$elements of U not in $A\}$

$= \{0, 7, 8, 9, 10\}$

and, $B' = \{$element of U not in $B\}$

$= \{0, 1, 2, 3, 9, 10\}$

$\therefore$ Intersection of A' and B' in $A' \cap B'$ contains all elements common to A' and B',

$\therefore A' \cap B' = \{0, 9, 10\}$ —————— (2)

From equation (1) and (2), it can be concluded that $(A \cup B)' = A' \cap B'$ ———— (3)

Thus, equation (3) represents the De Morgan's rule.