

(c) Matrix Multiplication: The condition of conformability, i.e., the necessary condition for matrix multiplication has to be satisfied for multiplication of two matrices (say A and B). The matrices A and B are conformable for multiplication in the form AB if the number of columns of the first matrix (A) has to be equal to the number of rows of the second matrix (B).

If matrix A is of dimension $(c \times d)$ and matrix B is of order $(d \times e)$, then multiplication of the two matrices in the form 'AB' can be defined, because the number of columns of the first matrix A (i.e., d) is equal to the number of rows of the second matrix B (i.e., d) and the order or dimension of the newly formed matrix will be $(c \times e)$. But, it is not possible to define BA, because the number of columns of the first matrix B (i.e., e) is not equal to the number of rows of the second matrix A (i.e., c).

In matrix multiplication, the first matrix is defined as 'lead' matrix and the second one is termed as 'lag' matrix.

Thus, if $AB = C$, the dimension of C will be equal to the number of rows of lead matrix (A) and number of columns of the lag matrix (B). & Thus, symbolically,

$$A_{(c \times d)} \cdot B_{(d \times e)} = C_{(c \times e)}$$

For example,

$$A_{(2 \times 3)} \cdot B_{(3 \times 2)} = C_{(2 \times 2)}$$

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{(2 \times 2)}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{(2 \times 2)}$, then

multiplication of the two matrices in both the forms (i.e., AB and BA) can be possible because it fulfills the condition of conformability and the newly formed matrix's (say C) order will be (2×2) .

$$\begin{aligned} \text{Thus, } A \cdot B &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{(2 \times 2)} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}_{(2 \times 2)} \\ &= \begin{bmatrix} (a_{11} \cdot b_{11} + a_{12} \cdot b_{21}) & (a_{11} \cdot b_{12} + a_{12} \cdot b_{22}) \\ (a_{21} \cdot b_{11} + a_{22} \cdot b_{21}) & (a_{21} \cdot b_{12} + a_{22} \cdot b_{22}) \end{bmatrix}_{(2 \times 2)} = C \end{aligned}$$

Similarly, $BA = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\begin{aligned} &= \begin{bmatrix} (b_{11} \cdot a_{11} + b_{12} \cdot a_{21}) & (b_{11} \cdot a_{12} + b_{12} \cdot a_{22}) \\ (b_{21} \cdot a_{11} + b_{22} \cdot a_{21}) & (b_{21} \cdot a_{12} + b_{22} \cdot a_{22}) \end{bmatrix}_{(2 \times 2)} \\ &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}_{(2 \times 2)} = C \end{aligned}$$

Accordingly, if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{(3 \times 3)}$ & $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{(3 \times 3)}$

$$\therefore AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{(3 \times 3)} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{(3 \times 3)} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_{(3 \times 3)}$$

$$= \begin{bmatrix} (a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}) & (a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32}) & (a_{11} \cdot b_{13} + a_{12} \cdot b_{23} + a_{13} \cdot b_{33}) \\ (a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}) & (a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + a_{23} \cdot b_{32}) & (a_{21} \cdot b_{13} + a_{22} \cdot b_{23} + a_{23} \cdot b_{33}) \\ (a_{31} \cdot b_{11} + a_{32} \cdot b_{21} + a_{33} \cdot b_{31}) & (a_{31} \cdot b_{12} + a_{32} \cdot b_{22} + a_{33} \cdot b_{32}) & (a_{31} \cdot b_{13} + a_{32} \cdot b_{23} + a_{33} \cdot b_{33}) \end{bmatrix}$$

$$= C$$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{(2 \times 3)}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{(3 \times 2)}$

$\therefore AB = C$ or, $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{(2 \times 3)} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{(3 \times 2)}$

$$= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}_{(2 \times 2)}$$

$$= \begin{bmatrix} (a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}) & (a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32}) \\ (a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}) & (a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + a_{23} \cdot b_{32}) \end{bmatrix}$$

Similarly, $BA = C$ or $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{(3 \times 2)} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{(2 \times 3)}$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_{(3 \times 3)}$$

$$= \begin{bmatrix} (b_{11} \cdot a_{11} + b_{12} \cdot a_{21}) & (b_{11} \cdot a_{12} + b_{12} \cdot a_{22}) & (b_{11} \cdot a_{13} + b_{12} \cdot a_{23}) \\ (b_{21} \cdot a_{11} + b_{22} \cdot a_{21}) & (b_{21} \cdot a_{12} + b_{22} \cdot a_{22}) & (b_{21} \cdot a_{13} + b_{22} \cdot a_{23}) \\ (b_{31} \cdot a_{11} + b_{32} \cdot a_{21}) & (b_{31} \cdot a_{12} + b_{32} \cdot a_{22}) & (b_{31} \cdot a_{13} + b_{32} \cdot a_{23}) \end{bmatrix}$$

Again if $A = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}_{(3 \times 1)}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \end{bmatrix}_{(1 \times 3)}$

then, $AB = C$ or $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}_{(3 \times 1)} \begin{bmatrix} b_{11} & b_{12} & b_{13} \end{bmatrix}_{(1 \times 3)}$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_{(3 \times 3)} = \begin{bmatrix} (a_{11} \cdot b_{11}) & (a_{11} \cdot b_{12}) & (a_{11} \cdot b_{13}) \\ (a_{21} \cdot b_{11}) & (a_{21} \cdot b_{12}) & (a_{21} \cdot b_{13}) \\ (a_{31} \cdot b_{11}) & (a_{31} \cdot b_{12}) & (a_{31} \cdot b_{13}) \end{bmatrix}_{(3 \times 3)}$$

$$BA = C \text{ or } \begin{bmatrix} b_{11} & b_{12} & b_{13} \end{bmatrix}_{(1 \times 3)} \cdot \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}_{(3 \times 1)}$$

$$= \begin{bmatrix} c_{11} \end{bmatrix}_{(1 \times 1)} = \begin{bmatrix} b_{11} \cdot a_{11} + b_{12} \cdot a_{21} + b_{13} \cdot a_{31} \end{bmatrix}_{(1 \times 1)}$$

If $A = \begin{bmatrix} a_{11} \end{bmatrix}_{(1 \times 1)}$ and $B = \begin{bmatrix} b_{11} \end{bmatrix}_{(1 \times 1)}$

then $AB = C$ or $\begin{bmatrix} a_{11} \end{bmatrix}_{(1 \times 1)} \cdot \begin{bmatrix} b_{11} \end{bmatrix}_{(1 \times 1)}$

$$= \begin{bmatrix} c_{11} \end{bmatrix}_{(1 \times 1)} = \begin{bmatrix} a_{11} \cdot b_{11} \end{bmatrix}_{(1 \times 1)}$$

and $BA = C$ or $\begin{bmatrix} b_{11} \end{bmatrix}_{(1 \times 1)} \cdot \begin{bmatrix} a_{11} \end{bmatrix}_{(1 \times 1)}$

$$= \begin{bmatrix} c_{11} \end{bmatrix}_{(1 \times 1)} = \begin{bmatrix} b_{11} \cdot a_{11} \end{bmatrix}_{(1 \times 1)}$$

Examples:

(a) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}_{(2 \times 2)}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}_{(2 \times 2)}$, find ~~the~~

AB and BA .

Solution: It is given that,

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}_{(2 \times 2)} \quad B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}_{(2 \times 2)}$$

Multiplication of matrices A and B can be possible, ~~because~~ in both the forms AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\therefore AB = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} (2 \cdot 1 + 3 \cdot 3) & (2 \cdot 4 + 3 \cdot 2) \\ (1 \cdot 1 + 4 \cdot 3) & (1 \cdot 4 + 4 \cdot 2) \end{bmatrix}_{(2 \times 2)}$$

$$= \begin{bmatrix} (2+9) & (8+6) \\ (1+12) & (4+8) \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} 11 & 14 \\ 13 & 12 \end{bmatrix}_{(2 \times 2)}$$

Similarly, $BA = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} (1 \cdot 2 + 4 \cdot 1) & (1 \cdot 3 + 4 \cdot 4) \\ (3 \cdot 2 + 2 \cdot 1) & (3 \cdot 3 + 2 \cdot 4) \end{bmatrix}_{(2 \times 2)}$

$$= \begin{bmatrix} (2+4) & (3+16) \\ (6+2) & (9+8) \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} 6 & 19 \\ 8 & 17 \end{bmatrix}_{(2 \times 2)}$$

(b) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$, find AB and BA .

Solution: It is given that

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{(2 \times 2)} \quad B = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}_{(2 \times 2)}$$

Multiplication of the given matrices A and B can be possible in both the forms AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{(2 \times 2)} \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} (2 \cdot 5 + 3 \cdot 1) & (2 \cdot 4 + 3 \cdot 1) \\ (4 \cdot 5 + 5 \cdot 1) & (4 \cdot 4 + 5 \cdot 1) \end{bmatrix}_{(2 \times 2)} \\ &= \begin{bmatrix} (10+3) & (8+3) \\ (20+5) & (16+5) \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} 13 & 11 \\ 25 & 21 \end{bmatrix}_{(2 \times 2)} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } BA &= \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}_{(2 \times 2)} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} (5 \cdot 2 + 4 \cdot 4) & (5 \cdot 3 + 4 \cdot 5) \\ (1 \cdot 2 + 1 \cdot 4) & (1 \cdot 3 + 1 \cdot 5) \end{bmatrix}_{(2 \times 2)} \\ &= \begin{bmatrix} (10+16) & (15+20) \\ (2+4) & (3+5) \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} 26 & 35 \\ 6 & 8 \end{bmatrix}_{(2 \times 2)} \end{aligned}$$

(c) If $A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}$, find AB & BA .

Solution: It is given that

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}_{(3 \times 3)} \quad \text{and} \quad B = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}_{(3 \times 3)}$$

Multiplication of the given matrices A and B can be possible in both the forms AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 3 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}_{(3 \times 3)} \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}_{(3 \times 3)} \\ &= \begin{bmatrix} (3 \cdot 3 + 0 \cdot 2 + 1 \cdot 2) & (3 \cdot 4 + 0 \cdot 1 + 1 \cdot 2) & (3 \cdot 2 + 0 \cdot 3 + 1 \cdot 1) \\ (2 \cdot 3 + 2 \cdot 2 + 3 \cdot 2) & (2 \cdot 4 + 2 \cdot 1 + 3 \cdot 2) & (2 \cdot 2 + 2 \cdot 3 + 3 \cdot 1) \\ (4 \cdot 3 + 1 \cdot 2 + 2 \cdot 2) & (4 \cdot 4 + 1 \cdot 1 + 2 \cdot 2) & (4 \cdot 2 + 1 \cdot 3 + 2 \cdot 1) \end{bmatrix}_{(3 \times 3)} \end{aligned}$$

$$= \begin{bmatrix} (9+0+2) & (12+0+2) & (6+0+1) \\ (6+4+6) & (8+2+6) & (4+6+3) \\ (12+2+4) & (6+1+4) & (8+3+2) \end{bmatrix} = \begin{bmatrix} 11 & 14 & 7 \\ 16 & 16 & 13 \\ 18 & 11 & 13 \end{bmatrix} \quad (3 \times 3)$$

Similarly, $BA = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix} \quad (3 \times 3)$

$$= \begin{bmatrix} (2 \cdot 0 + 4 \cdot 2 + 2 \cdot 4) & (3 \cdot 0 + 4 \cdot 2 + 2 \cdot 1) & (3 \cdot 1 + 4 \cdot 3 + 2 \cdot 2) \\ (2 \cdot 3 + 1 \cdot 2 + 3 \cdot 4) & (2 \cdot 0 + 1 \cdot 2 + 3 \cdot 1) & (2 \cdot 1 + 1 \cdot 3 + 3 \cdot 2) \\ (2 \cdot 3 + 2 \cdot 2 + 1 \cdot 4) & (2 \cdot 0 + 2 \cdot 2 + 1 \cdot 1) & (2 \cdot 1 + 2 \cdot 3 + 1 \cdot 2) \end{bmatrix} \quad (3 \times 3)$$

$$= \begin{bmatrix} (9+8+8) & (0+8+2) & (3+12+4) \\ (6+2+12) & (0+2+3) & (2+3+6) \\ (6+4+4) & (0+4+1) & (2+6+2) \end{bmatrix} = \begin{bmatrix} 25 & 10 & 19 \\ 20 & 5 & 11 \\ 14 & 5 & 10 \end{bmatrix} \quad (3 \times 3)$$

(2) If $A = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 2 & -1 \\ 5 & 4 & 2 \end{bmatrix}$ find AB & BA .

Solution: It is given that

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 2 & -1 \\ 5 & 4 & 2 \end{bmatrix}$$

Multiplication of the given matrices can be possible in both the forms AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\therefore AB = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 2 & 2 & -1 \\ 5 & 4 & 2 \end{bmatrix} = \begin{bmatrix} (1 \cdot 2 + (-3) \cdot 2 + 5 \cdot 5) & (1 \cdot 3 + (-3) \cdot 2 + 5 \cdot 4) & (1 \cdot 4 + (-3) \cdot (-1) + 5 \cdot 2) \\ (0 \cdot 2 + 2 \cdot 2 + 4 \cdot 5) & (0 \cdot 3 + 2 \cdot 2 + 4 \cdot 4) & (0 \cdot 4 + 2 \cdot (-1) + 4 \cdot 2) \\ (3 \cdot 2 + 4 \cdot 2 + 5 \cdot 5) & (3 \cdot 3 + 4 \cdot 2 + 5 \cdot 4) & (3 \cdot 4 + 4 \cdot (-1) + 5 \cdot 2) \end{bmatrix} \quad (3 \times 3)$$

$$= \begin{bmatrix} (2-6+25) & (3-6+20) & (4+3+10) \\ (0+4+20) & (0+4+16) & (0-2+8) \\ (6+8+25) & (9+8+20) & (12-4+10) \end{bmatrix} \quad (3 \times 3)$$

$$= \begin{bmatrix} 21 & 17 & 17 \\ 24 & 20 & 6 \\ 39 & 37 & 18 \end{bmatrix} \quad (3 \times 3)$$

Similarly, $BA = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 2 & -1 \\ 5 & 4 & 2 \end{bmatrix}_{(3 \times 3)} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix}_{(3 \times 3)}$

$$= \begin{bmatrix} (2 \cdot 1 + 3 \cdot 0 + 4 \cdot 3) & (2 \cdot (-3) + 3 \cdot 2 + 4 \cdot 4) & (2 \cdot 5 + 3 \cdot 4 + 4 \cdot 5) \\ (2 \cdot 1 + 2 \cdot 0 + (-1) \cdot 3) & (2 \cdot (-3) + 2 \cdot 2 + (-1) \cdot 4) & (2 \cdot 5 + 2 \cdot 4 + (-1) \cdot 5) \\ (5 \cdot 1 + 4 \cdot 0 + 2 \cdot 3) & (5 \cdot (-3) + 4 \cdot 2 + 2 \cdot 4) & (5 \cdot 5 + 4 \cdot 4 + 2 \cdot 5) \end{bmatrix}_{(3 \times 3)}$$

$$= \begin{bmatrix} (2+0+12) & (-6+6+16) & (10+12+20) \\ (2+0-3) & (-6+4-4) & (10+8-5) \\ (5+0+6) & (-15+8+8) & (25+16+10) \end{bmatrix}_{(3 \times 3)}$$

$$= \begin{bmatrix} 14 & 16 & 42 \\ -1 & -6 & 13 \\ 11 & 1 & 51 \end{bmatrix}_{(3 \times 3)}$$

(e) If $A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \end{bmatrix}_{(2 \times 3)}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}_{(3 \times 2)}$ find AB & BA .

Solution: It is given that

$$A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \end{bmatrix}_{(2 \times 3)} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}_{(3 \times 2)}$$

Multiplication of the given matrices A and B can be possible in both the form AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \end{bmatrix}_{(2 \times 3)} \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}_{(3 \times 2)} \\ &= \begin{bmatrix} (3 \cdot 1 + 1 \cdot (-2) + 5 \cdot 1) & (3 \cdot 3 + 1 \cdot 4 + 5 \cdot 0) \\ (2 \cdot 1 + 0 \cdot (-2) + 4 \cdot 1) & (2 \cdot 3 + 0 \cdot 4 + 4 \cdot 0) \end{bmatrix}_{(2 \times 2)} \\ &= \begin{bmatrix} (3-2+5) & (9+4+0) \\ (2-0+4) & (6+0+0) \end{bmatrix}_{(2 \times 2)} = \begin{bmatrix} 6 & 13 \\ 6 & 6 \end{bmatrix}_{(2 \times 2)} \end{aligned}$$

Similarly, $BA = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}_{(3 \times 2)} \begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \end{bmatrix}_{(2 \times 3)}$

$$= \begin{bmatrix} (1 \cdot 3 + 3 \cdot 2) & (1 \cdot 1 + 3 \cdot 0) & (1 \cdot 5 + 3 \cdot 4) \\ (-2 \cdot 3 + 4 \cdot 2) & (-2 \cdot 1 + 4 \cdot 0) & (-2 \cdot 5 + 4 \cdot 4) \\ (1 \cdot 3 + 0 \cdot 2) & (1 \cdot 1 + 0 \cdot 0) & (1 \cdot 5 + 0 \cdot 4) \end{bmatrix}_{(3 \times 3)} = \begin{bmatrix} (3+6) & (1+0) & (5+12) \\ (-6+8) & (-2+0) & (-10+16) \\ (3+0) & (1+0) & (5+0) \end{bmatrix}_{(3 \times 3)}$$

$$= \begin{bmatrix} 9 & 1 & 17 \\ 2 & -2 & 6 \\ 3 & 1 & 5 \end{bmatrix}_{(3 \times 3)}$$

(6) If $A = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}$, find AB and BA .

Solution: It is given that

$$A = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{(1 \times 3)} \quad \text{and} \quad B = \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}_{(3 \times 1)}$$

Multiplication of the given matrices A and B can be possible in both the forms AB and BA , because it fulfills the condition of conformability for matrix multiplication.

$$\therefore AB = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{(1 \times 3)} \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}_{(3 \times 1)} = \begin{bmatrix} 2 \cdot 5 + 3 \cdot 6 + 1 \cdot 3 \end{bmatrix}_{(1 \times 1)} = \begin{bmatrix} 31 \end{bmatrix}_{(1 \times 1)}$$

$$\text{Similarly } BA = \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}_{(3 \times 1)} \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{(1 \times 3)} = \begin{bmatrix} (5 \cdot 2) & (5 \cdot 3) & (5 \cdot 1) \\ (6 \cdot 2) & (6 \cdot 3) & (6 \cdot 1) \\ (3 \cdot 2) & (3 \cdot 3) & (3 \cdot 1) \end{bmatrix}_{(3 \times 3)} = \begin{bmatrix} 10 & 15 & 5 \\ 12 & 18 & 6 \\ 6 & 9 & 3 \end{bmatrix}_{(3 \times 3)}$$

(8) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 & 6 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, verify whether AB and BA will exist and if exist find the value.

Solution: It is given that

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \end{bmatrix}_{(2 \times 3)} \quad \text{and} \quad B = \begin{bmatrix} 5 & 6 & 6 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}_{(3 \times 3)}$$

Multiplication of the given matrices A and B can be possible in the form AB , because it fulfills the condition of conformability for matrix multiplication, i.e., number of columns of the first matrix A (i.e., 3) is equal to the number of rows of the second matrix B (i.e., 3).

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \end{bmatrix}_{(2 \times 3)} \begin{bmatrix} 5 & 6 & 6 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}_{(3 \times 3)} \\ &= \begin{bmatrix} (3 \cdot 5 + 4 \cdot 2 + 2 \cdot 3) & (3 \cdot 6 + 4 \cdot 3 + 2 \cdot 4) & (3 \cdot 6 + 4 \cdot 1 + 2 \cdot 2) \\ (5 \cdot 5 + 6 \cdot 2 + 3 \cdot 3) & (5 \cdot 6 + 6 \cdot 3 + 3 \cdot 4) & (5 \cdot 6 + 6 \cdot 1 + 3 \cdot 2) \end{bmatrix}_{(2 \times 3)} \\ &= \begin{bmatrix} (15 + 8 + 6) & (18 + 12 + 8) & (18 + 4 + 4) \\ (25 + 12 + 9) & (30 + 18 + 12) & (30 + 6 + 6) \end{bmatrix}_{(2 \times 3)} \\ &= \begin{bmatrix} 29 & 38 & 26 \\ 46 & 60 & 42 \end{bmatrix}_{(2 \times 3)} \end{aligned}$$

Multiplication of matrices A and B in the form BA will not be possible because it does not fulfill the condition of conformability, i.e., number of columns of the first matrix B (i.e., 1) is not equal to the number of rows of the second matrix A (i.e., 3). So, BA does not exist for the above given matrices.

(1) If $A = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 3 & 4 \\ 1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, verify whether

AB and BA exist and if exist find the value.

Solution: It is given that

$$A = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 3 & 4 \\ 1 & 1 & 3 \end{bmatrix}_{(3 \times 3)} \quad B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{(3 \times 1)}$$

Multiplication of the given matrices A and B in the form AB will be possible, because it fulfills the condition of conformability, i.e., number of columns of the first matrix A (i.e., 3) is equal to the number of rows of the second matrix B (i.e., 3).

$$\therefore AB = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 3 & 4 \\ 1 & 1 & 3 \end{bmatrix}_{(3 \times 3)} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{(3 \times 1)}$$

$$= \begin{bmatrix} (4 \cdot 2 + 5 \cdot 1 + 1 \cdot 3) \\ (2 \cdot 2 + 3 \cdot 1 + 4 \cdot 3) \\ (1 \cdot 2 + 1 \cdot 1 + 3 \cdot 3) \end{bmatrix}_{(3 \times 1)} = \begin{bmatrix} 16 \\ 19 \\ 12 \end{bmatrix}_{(3 \times 1)}$$

Multiplication of the given matrices A and B in the form BA will not be possible, because it does not fulfill the condition of conformability, i.e., number of columns of the first matrix B (i.e., 1) is not equal to the number of rows of the second matrix A (i.e., 3). So, BA does not exist for the above given matrices.