ASSUMPTION OF GENERAL LINEAR REGRESSION MODEL :

(1) **Assumption 1 :**

$\mathcal{E}(U) = 0$, where U is a $n \times 1$ column matrice and 0 is a $n \times 1$ column null matrix.

$$\therefore \mathcal{E}(U) = 0$$

$$\Rightarrow \mathcal{E}\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{pmatrix}_{n \times 1} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{n \times 1}$$

$$\Rightarrow \begin{pmatrix} \mathcal{E}(U_1) \\ \mathcal{E}(U_2) \\ \vdots \\ \mathcal{E}(U_n) \end{pmatrix}_{n \times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1}$$

(ii) **Assumption 2 :**

$\mathcal{E}(UU') = \sigma^2 I_n$; where σ^2 is a scaler and I_n is an identity matrix of a order $n \times n$.

$$\therefore \mathcal{E} = (UU') = \mathcal{E}\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}_{n \times 1} \begin{bmatrix} U_1 & U_2 & \cdots & U_n \end{bmatrix}_{1 \times n}$$

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$$= E\begin{bmatrix} U_1^2 & U_1U_2 & --- & U_1U_n \\ U_1U_2 & U_2^2 & --- & U_2U_n \\ \vdots & & & \\ U_1U_n & U_2U_n & --- & U_n^2 \end{bmatrix}_{n\times n}$$

$$= \begin{bmatrix} E(U_1^2) & E(U_1U_2) & ---- & E(U_1U_n) \\ E(U_1U_2) & E(U_2^2) & --- & E(U_2U_n) \\ E(U_1U_n) & E(U_2U_3) & --- & E(U_2^2) \end{bmatrix}_{n\times n}$$

$$= \begin{bmatrix} 6^2 & 0 & --- & 0 \\ 0 & 6^2 & --- & 0 \\ 0 & 0 & --- & 6^2 \end{bmatrix}_{n\times n}$$

$$= 6^2\begin{bmatrix} 1 & 0 & --- & 0 \\ 0 & 1 & --- & 0 \\ 0 & 0 & --- & 1 \end{bmatrix}_{n\times n}$$

$$= 6^2 I_n$$

$$\therefore E(UU') = 6^2 I_n$$

(iii) Assumption (iii):

$$E(U_i^2) = 6^2$$

where $i = 1, 2 --- n$; this assumtion is known as homoscedasticity

(iv) Assumption (iv):-

$E(U_i U_j) = 0$; this means that the random term of different observations are independent. This assumption is known as assumption of no autocorrelation.

(v) Assumption:- $U \sim N(0\ 6^2 I_n)$ that means the

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random disturbance term follows the normal distribution.

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ESTIMATION OF PARAMETERS BY OLS METHOD IN CASE OF MULTIPLE LINEAR REGRESSION

Let us consider following K variable linear regression model.

$$Y_t = B_1 + B_2 X_2 + B_3 X_3 - - - - - + B_K X_{Kt} + U_t \rightarrow (1)$$

where $t = 1, 2, ---- n$

Now, to estimate the unknown parameters B_1, $B_2 ---- B_K$ let us first right the estimated regression model.

$$\hat{Y}_t = \hat{B}_1 + \hat{B}_2 X_2 + \hat{B}_3 X_3 + ---- \hat{B}_K X_{Kt} + \rightarrow (2)$$

where, $\hat{B}_1, \hat{B}_2, \hat{B}_3 ---- \hat{B}_K$ are the estimated values of $B_1, B_2, B_3 --- B_K$ respectively.

The main Principle of OLS method is to estimate the unknown parameters in such a way that the sum square of the residuals is least or minimum.]

[The sum square of the residuals is given by ______

$$\Sigma \hat{U}_t^2 = \text{ where}$$

$$\hat{U}_t = Y_t - \hat{Y}_t$$

$$= Y_t - \hat{B}_1 - \hat{B}_2 X_{2t} - \hat{B}_3 X_{3t} ---- - \hat{B}_K X_{Kt}$$

$$\Sigma \hat{U}_t^2 = \Sigma \left(Y_t - \hat{B}_1 - \hat{B}_2 X_{2t} - \hat{B}_3 X_{3t} --- - \hat{B}_K X_{Kt}\right)^2$$

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For minimisation of first order condition is

$$\frac{\partial \Sigma \hat{u}_t^2}{\partial \hat{\beta}_1} = 0, \quad \frac{\partial \Sigma \hat{U}_t^2}{\partial \hat{\beta}_2} = 0 \text{ --------- } \frac{\partial \Sigma \hat{U}_t^2}{\partial \hat{\beta}_K} = 0$$

$$\Rightarrow \frac{\partial \Sigma \hat{U}_t^2}{\partial \hat{\beta}_1} = 2\Sigma(Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_{2t} \text{ ----- } \hat{\beta}_K X_{Kt})(-1) = 0$$

$$\Rightarrow \frac{\partial \Sigma \hat{U}_t^2}{\partial \hat{\beta}_2} = 2\Sigma(Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_{2t} \text{ ---- } \hat{\beta}_K X_{Kt})(-X_{2t}) = 0 \quad (3)$$

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Last,

$$\Rightarrow \frac{\partial \Sigma \hat{U}_t^2}{\partial \hat{\beta}_K} = 2\Sigma(Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_{2t} \text{ --- } \hat{\beta}_K X_{Kt})(-X_{Kt}) = 0$$

The set of equation can be written as ——

$$\Rightarrow n\hat{\beta}_1 + \hat{\beta}_2 \Sigma X_{2t} + \hat{\beta}_3 \Sigma X_{3t} \text{ ---- } \hat{\beta}_K \Sigma X_{Kt} = \Sigma Y_t$$

$$\Rightarrow \hat{\beta}_1 \Sigma X_{2t} + \hat{\beta}_2 \Sigma X_{2t}^2 + \hat{\beta}_3 \Sigma X_{2t} X_{3t} + \text{ ----- } + \hat{\beta}_K \Sigma X_{2t} X_{Kt} = \Sigma X_{2t} Y_t$$

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$$\Rightarrow \hat{\beta}_1 \Sigma X_{Kt} + \hat{\beta}_2 \Sigma X_{2t} X_{Kt} + \hat{\beta}_3 \Sigma X_{3t} X_{Kt} + \text{ --- } \text{ ---- } \hat{\beta}_K \Sigma X_{Kt}^2 = \Sigma X_{Kt} Y_t.$$

The set of equation (4) can be arranged in the following matrix form ——

$$\begin{bmatrix} n & \Sigma X_{2t} & \Sigma X_{3t} & \cdots & \Sigma X_{Kt} \\ \Sigma X_{2t} & \Sigma X_{2t}^2 & \Sigma X_{2t} X_{3t} & \cdots & \Sigma X_{2t} X_{Kt} \\ \vdots & & & & \vdots \\ \Sigma X_{Kt} & \Sigma X_{2t} X_{Kt} & \Sigma X_{3t} X_{Kt} & \cdots & \Sigma X_{Kt}^2 \end{bmatrix}_{K \times K} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_K \end{bmatrix}_{K \times 1}$$

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$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{21} & x_{22} & \cdots & x_{2n} \\ - & - & - & - \\ - & - & - & - \\ x_{k1} & x_{k2} & \cdots & x_{kn} \end{bmatrix}_{k\times n} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n\times 1}$$

$$= \left\{ \begin{bmatrix} 1 & 1 & 1 \\ x_{21} & x_{22} & x_{2n} \\ - & - & - \\ x_{k1} & x_{k2} & x_{kn} \end{bmatrix}_{k\times n} \times \begin{bmatrix} 1 & x_{21} & \cdots & x_{k1} \\ 1 & x_{22} & \cdots & x_{k2} \\ - & - & - & - \\ 1 & x_{2n} & & x_{kn} \end{bmatrix}_{n\times k} \right\}$$

$$\times \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}_{k\times 1} = \begin{bmatrix} 1 & 1 & 1 \\ x_{21} & x_{22} & x_{2n} \\ - & - & - \\ x_{k1} & x_{k2} & \cdots\ x_{kn} \end{bmatrix}_{k\times n} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n\times 1}$$

$$\Rightarrow X'X\hat{\beta} = X'Y$$

multiplying both side of the above equation by $(X'X)^{-1}$

$$\Rightarrow (X'X)(X'X)^{-1}\hat{\beta} = (X'X)^{-1}X'Y$$

$$\Rightarrow \hat{\beta} = (X'X)^{-1}X'Y.$$